AQFT: problem sheet 4

1. Off-shell BRST transformations are

c is in the adjoint

$$\delta A^a_\mu = \varepsilon (D_\mu c)^a = \varepsilon (\partial_\mu c^a - g f^{abc} c^b A^c_\mu)$$

$$\delta B^a = 0$$

$$\delta \bar{c}^a = -\varepsilon B^a$$

$$\delta c^a = -\tfrac{1}{2} g \varepsilon f^{abc} c^b c^c$$

for some constant Grassmann parameter, and where

$$D_\mu = \partial_\mu - ig A^a_\mu T^a$$

Let us check nilpotency using two distinct parameters

$$\delta_2 \delta_1 A^a_\mu = \delta_2 \left[\varepsilon_1 (\partial_\mu c^a - g f^{abc} c^b A^c_\mu) \right]$$

$$= \varepsilon_1 (\partial_\mu \delta_2 c^a - g f^{abc} \delta_2 c^b A^c_\mu - g f^{abc} c^b \delta_2 A^c_\mu)$$

$$= \varepsilon_1 (D_\mu \delta_2 c^a - g f^{abc} c^b \delta_2 A^c_\mu)$$

$$= \varepsilon_1 \left[D_\mu \left(-\tfrac{1}{2} g \varepsilon_2 f^{abc} c^b c^c\right) - g f^{abc} \varepsilon_2 (D_\mu c)^c c^b \right]$$

$$= \varepsilon_1 \varepsilon_2 g \left[-\tfrac{1}{2} f^{abc} (D_\mu c)^b c^c - \tfrac{1}{2} f^{abc} c^b (D_\mu c)^c - f^{abc} (D_\mu c)^c c^b \right]$$

$$= 0$$

$$\delta_2 \delta_1 B = \delta_2 (0) = 0$$

$$\delta_2 \delta_1 \bar{c}^a = \delta_2 (-\varepsilon_1 B^a) = -\varepsilon_1 \delta_2 B^a = 0$$

$$s_2 s_1 c^a = s_2\left(-\tfrac{i}{2} g\, \varepsilon_1 f^{abc} c^b c^c\right)$$

$$= -\tfrac{i}{2}\varepsilon_1 g f^{abc}\left[(s_2 c^b) c^c + c^b (s_2 c^c)\right]$$

$$= \left(-\tfrac{i}{2} g\right)^2 \varepsilon_1 f^{abc}\left(\varepsilon_2 f^{bde} c^d c^e c^c + c^b \cdot \varepsilon_2 f^{cde} c^d c^e\right)$$

$$= \left(-\tfrac{i}{2} g\right)^2 \varepsilon_1 \varepsilon_2 \left(f^{acb} f^{cde} c^d c^e c^b - f^{abc} f^{cde} c^b c^d c^e\right)$$

$$= -\left(\tfrac{g}{2}\right)^2 \varepsilon_1 \varepsilon_2 f^{abc} f^{cde} \underbrace{c^b c^d c^e}_{\text{totally anti-sym.}} = 0$$

Have shown this for fundamental fields, but also should verify composite fields $O(\phi^i)$, collectively denoting the fundamental fields by ϕ^i:

$$s_2 s_1 O = s_2\left(s_1 \phi^i \frac{\delta O}{\delta \phi^i}\right)$$

$$= \cancel{s_2 s_1 \phi^i} \frac{\delta O}{\delta \phi^i} + s_1 \phi^i \, s_2 \phi^j \frac{\delta^2 O}{\delta \phi^j \delta \phi^i}$$

write $s = \varepsilon s_B$ in terms of the slavnov operator

$$= \varepsilon_1 (s_B \phi^i)\, \varepsilon_2 (s_B \phi^j) \frac{\delta^2 O}{\delta \phi^j \delta \phi^i}$$

$$= (-)^{|\phi^i|} \varepsilon_1 \varepsilon_2 (s_B \phi^i)(s_B \phi^j) \frac{\delta^2 O}{\delta \phi^j \delta \phi^i}$$

Now, $s_B \phi$ has opposite $\mathbb{Z}_2$-grading to ϕ. Hence, when both ϕ^i and ϕ^j are even/odd, this vanishes! Only the cross terms remain

$$= \varepsilon_1 \varepsilon_2 \Bigg(\sum_{\phi^i \text{ even}} \sum_{\phi^j \text{ odd}} (s_B \phi^i)(s_B \phi^j) \frac{\delta^2 O}{\delta \phi^j \delta \phi^i}$$

$$- \sum_{\phi^i \text{ odd}} \sum_{\phi^j \text{ even}} (s_B \phi^i)(s_B \phi^j) \frac{\delta^2 O}{\delta \phi^j \delta \phi^i} \Bigg)$$

$$= \varepsilon_1 \varepsilon_2 \sum_{\phi^i \text{ even}} \sum_{\phi^j \text{ odd}} \Bigg[(s_B \phi^i)(s_B \phi^j) \frac{\delta^2 O}{\delta \phi^j \delta \phi^i}$$

$$- (s_B \phi^j)(s_B \phi^i) \frac{\delta^2 O}{\delta \phi^i \delta \phi^j} \Bigg]$$

$= 0$

so δ is nilpotent.

2. Check that

$$s_B \left[-\bar{c}^a \partial^\mu A^a_\mu - \tfrac{1}{2}\xi \bar{c}^a B^a \right]$$

$$= -(s_B \bar{c}^a)\partial^\mu A^a_\mu + \bar{c}^a \partial^\mu (s_B A^a_\mu) - \tfrac{1}{2}\xi (s_B \bar{c}^a) B^a$$

$$+ \tfrac{1}{2}\xi \bar{c}^a (s_B B^a)$$

$$= B^a \partial^\mu A^a_\mu + \bar{c}^a \partial^\mu (D_\mu c^a) - \tfrac{1}{2}\xi(-B^a) B^a$$

$$= B^a \partial^\mu A^a_\mu + \bar{c}^a \partial^\mu (D_\mu c^a) + \tfrac{1}{2}\xi B^a B^a$$

then

$$\int d^D x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + s_B \left(-\bar{c}^a \partial^\mu A^a_\mu - \tfrac{1}{2}\xi \bar{c}^a B^a \right) \right]$$

$$= \int d^D x \left\{ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\partial^\mu \bar{c}^a)(D_\mu c^a) + B^a \partial^\mu A^a_\mu \right.$$

$$\left. + \tfrac{1}{2}\xi B^a B^a \right\}$$

which is indeed

$= S_{BRST}$

3. Check variation of action

$\delta_B S_{BRST}$

$$= \int d^D x \left[-\tfrac{1}{4} s_B (F_{\mu\nu} F^{\mu\nu}) + s_B^2 (\dots) \right]$$

Now, the BRST variation is just like a gauge transformation so $s_B F_{\mu\nu} = 0$, and s_B is also nilpotent, so

$= 0$

4. Picking Coulomb gauge, just need to map $\mu \to i$

$$S_{BRST,\ \text{Coulomb}} = \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + s_B \left(-\bar{c}^a \partial^i A_i^a - \frac{1}{2} \xi \bar{c}^a B^a \right) \right]$$

$$= \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\partial^i \bar{c}^a)(D_i c^a) + B^a \partial^i A_i^a + \frac{1}{2} \xi B^a B^a \right]$$

5. Integrate out B

$$Z = \int \mathcal{D}A\, \mathcal{D}B\, \mathcal{D}c\, \mathcal{D}\bar{c}\; e^{iS_{\text{off}}} \sim \int \mathcal{D}A\, \mathcal{D}c\, \mathcal{D}\bar{c}\; e^{iS_{\text{on}}}$$

Concentrating on the terms involving B,

$$\int \mathcal{D}B \exp\left[i \int d^D x \left(\frac{1}{2} \xi B^a B^a + B^a \partial^i A_i^a \right) \right]$$

$$= \int \mathcal{D}B \exp\left[i \int d^D x\, \frac{1}{2} \xi \left(B^a B^a + 2 \frac{1}{\xi} B^a \partial^i A_i^a \right) \right]$$

$$= \int \mathcal{D}B \exp\left[i \int d^D x\, \frac{1}{2} \xi \left(\left(B^a + \frac{1}{\xi} \partial^i A_i^a \right)^2 - \left(\frac{1}{\xi} \partial^i A_i^a \right)^2 \right) \right]$$

$$\sim \exp\left[i \int d^D x \left(-\frac{1}{2\xi} (\partial^i A_i^a)^2 \right) \right]$$

hence

$$S_{BRST,\ \text{Coulomb}}^{\text{on-shell}} = \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (\partial^i \bar{c}^a)(D_i c^a) - \frac{1}{2\xi} (\partial^i A_i^a)^2 \right]$$

Note: This is equivalent to the saddle-point approximation, i.e. using the equations of motion of B!

6. Define effective action as always

$$e^{iW[J,\eta,\bar\eta]} = Z[J,\eta,\bar\eta]$$

$$= \int DA\, Dc\, D\bar c \; \exp\Big[i S^{\text{on-shell}}_{\text{BRST, coulomb}}$$

$$+ i \int d^D x \left(J^a_\mu A^{a\mu} + \bar\eta^a c^a + \bar c^a \eta^a \right)\Big]$$

In the free-field limit $g=0$, we are left with the quadratic terms

$$S^{\text{on}}_{\text{BRST, c}} \supset \int d^D x \Big[-\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 - \frac{1}{2\xi}\left(\partial^i A^a_i\right)^2$$

$$- (\partial^i \bar c^a)(\partial_i c^a) \Big]$$

$$= \int d^D x \Big[-\frac{1}{2}\left(\partial_\mu A^a_\nu\right)^2 + \frac{1}{2}\left(\partial_\mu A^a_\nu\right)\left(\partial^\nu A^{a\mu}\right)$$

$$- \frac{1}{2\xi}\left(\partial^i A^a_i\right)^2 - (\partial^i \bar c^a)(\partial_i c^a) \Big]$$

$$= \int d^D x \Big[\frac{1}{2} A^a_\mu \Box A^{a\mu} + \frac{1}{2}\left(\partial_\mu A^{a\mu}\right)^2 - \frac{1}{2\xi}\left(\partial^i A^a_i\right)^2$$

$$- (\partial^i \bar c^a)(\partial_i c^a) \Big]$$

$$= \int d^D x \Big[\frac{1}{2} A^a_\mu \Box A^{a\mu} + \frac{1}{2}\left(\partial_0 A^{a0}\right)^2 + \left(\partial_0 A^{a0}\right)\left(\partial_i A^{ai}\right)$$

$$+ \frac{1}{2}\left(1 - \frac{1}{\xi}\right)\left(\partial^i A^a_i\right)^2 - (\partial^i \bar c^a)(\partial_i c^a) \Big]$$

Can do the path integral because it is just a Gaussian, but easier to use saddle point approximation, which is exact here. Denoting the fields collectively by ϕ, and the classical solutions by ϕ_{cl}, then we can expand the sourced actions as

$$S[\phi] = S[\bar\phi] + \cancel{\int d^D x \left.\frac{\delta S[\phi]}{\delta\phi(x)}\right|_{\phi=\phi_{cl}} \phi(x)}$$

$$+ \frac{1}{2}\int d^D x \int d^D y \left.\frac{\delta^2 S[\phi]}{\delta\phi(x)\,\delta\phi(y)}\right|_{\phi=\phi_{cl}} \phi(x)\,\phi(y))$$

for quadratic S - this is Gaussian and can be evaluated. Further, $\delta^2 S/\delta\phi^2$ will be independent of ϕ, so all sources will be $S[\bar\phi]$. This is what we need to compute!

For us, the classical equations of motion are

$$\frac{\delta S}{\delta A^{a0}} = -\Box A^{a0} - \partial_0^2 A^{a0} - \partial_0 \partial_i A^{ai} - J^{a0}$$

$$= -\partial_i \partial^i A^{a0} - \partial_0 \partial_i A^{ai} - J^{a0} = 0$$

$$\frac{\delta S}{\delta A^{ai}} = \Box A^{ai} - \partial_i \partial_0 A^{a0} - \left(1 - \frac{1}{\xi}\right) \partial_j \partial^j A^{ai} + J^{ai} = 0$$

$$\frac{\delta S}{\delta c^a} = -\partial_i \partial^i \bar c^a - \bar\eta^a = 0$$

$$\frac{\delta S}{\delta \bar c^a} = \partial_i \partial^i c^a + \eta^a = 0$$

The classical solutions for the ghosts are simply

$$c^a_{cl.} = -\frac{\eta^a}{\vec\nabla^2} \qquad \bar c^a_{cl} = -\frac{\bar\eta^a}{\vec\nabla^2}$$

and we need to work harder for the A's. To solve the A^0-equation, we need $\partial_i A^i$. To find this take a ∂_i derivative of the A^i-equation

$$0 = \Box \partial_i A^{ai} - \nabla^2 \partial_0 A^{a0} - \left(1 - \frac{1}{\xi}\right) \vec\nabla^2 \partial_i A^i + \partial_i J^{ai}$$

$$= \left(\Box - \left(1 - \frac{1}{\xi}\right)\vec\nabla^2\right) \partial_i A^{ai} - \nabla^2 \partial_0 A^{a0} + \partial_i J^{ai}$$

Hence

$$\partial_i A^{ai} = \frac{\nabla^2 \partial_0 A^{a0} - \partial_i J^{ai}}{\Box - \left(1 - \frac{1}{\xi}\right)\vec\nabla^2}$$

Plugging this into the A^0-equation

$$0 = -\underline{\vec{\nabla}^2 A^{a0}} - \frac{\partial_0}{\Box - (1 - 1/\xi)\vec{\nabla}^2}\left(\underline{\partial_0 \vec{\nabla}^2 A^{a0}} - \partial_i J^{ai}\right)$$

$$- J^{a0}$$

$$= \left(-1 - \frac{\partial_0^2}{\Box - (1 - 1/\xi)\vec{\nabla}^2}\right)\vec{\nabla}^2 A^{a0}$$

$$+ \frac{\partial_0 \partial_i}{\Box - (1 - 1/\xi)\vec{\nabla}^2} J^{ai} - J^{a0}$$

so that

$$A^{a0}_{cl} = -\left(1 + \frac{\partial_0^2}{\Box - (1 - 1/\xi)\vec{\nabla}^2}\right)^{-1} \frac{1}{\vec{\nabla}^2}$$

$$\left(-\frac{\partial_0 \partial_i}{\Box - (1 - 1/\xi)\vec{\nabla}^2} J^{ai} + J^{a0}\right)$$

$$= -\left(\frac{\cancel{\Box} - (\cancel{1} - 1/\xi)\vec{\nabla}^2 + \cancel{\partial_0^2}}{\Box - (1 - 1/\xi)\vec{\nabla}^2}\right)^{-1} \cdot \frac{1}{\nabla^2}\left(-\frac{\partial_0 \partial_i}{\Box - (1 - 1/\xi)\nabla^2} J^{ai}\right.$$

$$\left. + J^{a0}\right)$$

$$= -\frac{\Box - (1 - 1/\xi)\vec{\nabla}^2}{\vec{\nabla}^2/\xi} \cdot \frac{1}{\vec{\nabla}^2}\left(-\frac{\partial_0 \partial_i}{\Box - (1 - 1/\xi)\nabla^2} J^{ai} + J^{a0}\right)$$

$$= \xi \frac{\partial_0 \partial_i}{\vec{\nabla}^4} J^{ai} - \xi \frac{\Box - (1 - 1/\xi)\vec{\nabla}^2}{\vec{\nabla}^4} J^{a0}$$

Then plugging this back into the A^i equation

$$0 = \Box A^{ai} - \partial_i \partial_0 A^{a0} - \left(1 - \frac{1}{\xi}\right)\vec{\nabla}^2 A^{ai} + J^{ai}$$

$$= \left(\Box - (1 - 1/\xi)\vec{\nabla}^2\right) A^{ai} + J^{ai} - \partial_i \partial_0 A^{a0}$$

Therefore

$$A^{ai}_{cl} = \frac{1}{\Box - (1 - 1/\xi)\vec{\nabla}^2}\left(-J^{ai} + \partial_i \partial_0 A^{a0}\right)$$

$$= \frac{1}{\square - (1 - 1/\xi)\vec{\nabla}^2} \left(-J^{ai} + \partial_i \partial_0 \left(\xi \frac{\partial_0 \partial_j}{\vec{\nabla}^4} J^{aj} - \frac{\xi \square - (\xi - 1)\vec{\nabla}^2}{\vec{\nabla}^4} J^{a0} \right)\right)$$

$$= \frac{1}{\square - (1 - 1/\xi)\vec{\nabla}^2} \left(-J^{ai} + \xi \frac{\partial_0^2 \partial_i \partial_j}{\vec{\nabla}^4} J^{aj} \right)$$

$$+ \frac{1}{\square - (1 - 1/\xi)\vec{\nabla}^2} \cdot \partial_i \partial_0 \cdot - \frac{\xi \square - (\xi - 1)\vec{\nabla}^2}{\vec{\nabla}^4} J^{a0}$$

$$= \frac{1}{\square - (1 - 1/\xi)\vec{\nabla}^2} \left(-J^{ai} + \xi \frac{\partial_0^2 \partial_i \partial_j}{\vec{\nabla}^4} J^{aj} \right)$$

$$- \xi \frac{\partial_0 \partial_i}{\vec{\nabla}^4} J^{a0}$$

we then see that the free generating functional is

$$Z_0[J, \eta, \bar{\eta}] = \exp\left(i S^{on}_{BEST, c}[A_{cl}, c_{cl}, \bar{c}_{cl}] \right.$$

$$\left. + i \int d^D x \left[J^a_\mu A^{a\mu}_{cl} + \bar{\eta}^a c^a_{cl} - \bar{c}^a_{cl} \eta^a \right] \right) Z_0[0,0,0]$$

$$= \exp\left[i \int d^D x \left(-\tfrac{1}{2} A^{a0}_{cl} \square A^{a0}_{cl} + \tfrac{1}{2} A^{ai}_{cl} \square A^{ai}_{cl} + \tfrac{1}{2} (\partial_0 A^{a0}_{cl})^2 \right.\right.$$

$$\left.\left. + (\partial_0 A^{a0}_{cl})(\partial_i A^{ai}_{cl}) + \tfrac{1}{2}\left(1 - \tfrac{1}{\xi}\right)(\partial^i A^a_{cl\, i})^2 - (\partial_i \bar{c}^a_{cl})(\partial^i c^a_{cl}) \right)\right]$$

$$Z_0[0,0,0]$$

using the equations of motion

$$= \exp\left[i \int d^D x \left(\tfrac{1}{2} A^{a0}_{cl} \cdot J^{a0} + \tfrac{1}{2} A^{ai}_{cl} \cdot - J^{ai} \right.\right.$$

$$+ \tfrac{1}{2} \bar{c}^a \cdot - \eta^a + \tfrac{1}{2}(-\bar{\eta}^a) c^a - J^{a0} A^{a0}_{cl} + J^{ai} A^{ai}_{cl}$$

$$\left.\left. + \bar{\eta}^a c^a_{cl} - \bar{c}^a_{cl} \eta^a \right)\right] Z_0[0,0,0]$$

$$= \exp\left[\frac{i}{2} \int d^D x \left(-A^{a0}_{cl} J^{a0} + J^{ai} A^{ai}_{cl} + \bar{\eta}^a c^a_{cl} \right.\right.$$

$$\left.\left. + \bar{c}^a_{cl} \eta^a \right)\right] Z_0[0,0,0]$$

$$= \exp\left[\frac{i}{2}\int d^Dx\left(-\left(\xi\frac{\partial_0\partial_i}{\vec\nabla^4}J^{ai}-\xi\frac{\square-(1-1/\xi)\vec\nabla^2}{\vec\nabla^4}J^{a0}\right)J^{a0}\right.\right.$$

$$+J^{ai}\left(\frac{1}{\square-(1-1/\xi)\vec\nabla^2}\left(-J^{ai}+\xi\frac{\partial_0^2\partial_i\partial_j}{\vec\nabla^4}J^{aj}\right)-\xi\frac{\partial_0\partial_i}{\vec\nabla^4}J^{a0}\right)$$

$$\left.\left.+\bar\eta^a\left(-\frac{\eta^a}{\vec\nabla^2}\right)+\left(-\frac{\bar\eta^a}{\vec\nabla^2}\right)\eta^a\right)\right]Z_0[0,0,0]$$

$$= \exp\left[\frac{i}{2}\int d^Dx\left(\xi\frac{\square-(1-1/\xi)\vec\nabla^2}{\vec\nabla^4}J^{a0}J^{a0}\right.\right.$$

$$+J^{ai}\frac{1}{\square-(1-1/\xi)\vec\nabla^2}\left(-J^{ai}+\xi\frac{\partial_0^2\partial_i\partial_j}{\vec\nabla^4}J^{aj}\right)$$

$$\left.\left.-2\xi\frac{\partial_0\partial_i}{\vec\nabla^4}J^{a0}J^{ai}-2\frac{1}{\vec\nabla^2}\bar\eta^a\eta^a\right)\right]Z_0[0,0,0]$$

For the effective action, we therefore find

$$W[J,\eta,\bar\eta]-W[0,0,0]$$

$$=\int d^Dx\left(\frac{1}{2}J^{a0}\cdot\xi\frac{\square-(1-1/\xi)\vec\nabla^2}{\vec\nabla^4}J^{a0}\right.$$

$$+\frac{1}{2}J^{ai}\frac{-\delta^{ij}+\xi\frac{\partial_0^2\partial_i\partial_j}{\vec\nabla^4}}{\square-(1-1/\xi)\vec\nabla^2}J^{aj}-J^{a0}\cdot\xi\frac{\partial_0\partial_i}{\vec\nabla^4}J^{ai}$$

$$\left.-\bar\eta^a\frac{1}{\vec\nabla^2}\eta^a\right)$$

7. Just functionally differentiate the effective action

$$\langle A_0^a(x_1)A_0^b(x_2)\rangle$$

$$=\left(\frac{1}{i}\frac{\delta}{\delta J^{a0}(x_1)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J^{b0}(x_2)}\right)iW[J,\eta,\bar\eta]\Big|_{J,\eta,\bar\eta=0}$$

$$=-i\xi\frac{\square-(1-1/\xi)\vec\nabla^2}{\vec\nabla^4}\delta^{ab}\delta^{(D)}(x_1-x_2)$$

$$\langle A_0^a(x_1)A_i^b(x_2)\rangle$$

$$=\left(\frac{1}{i}\frac{\delta}{\delta J^{a0}(x_1)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J^{bi}(x_2)}\right)iW[J,\eta,\bar\eta]\Big|_{J,\eta,\bar\eta=0}$$

$$=+i\cdot\xi\frac{\partial_0\partial_i}{\vec\nabla^4}\delta^{ab}\delta^{(D)}(x_1-x_2)$$

$\langle A^a_i(x_1)\, A^b_j(x_2)\rangle$

$$= \left(\frac{1}{i}\frac{\delta}{\delta J^{ai}(x_1)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J^{bj}(x_2)}\right) iW[J,\eta,\bar\eta]\Big|_{J,\eta,\bar\eta=0}$$

$$= -i\cdot\frac{-\delta^{ij} + \xi\,\frac{\partial_0^2\,\partial_i\partial_j}{\nabla^4}}{\Box - (1-\xi)\nabla^2}$$

$\langle c^a(x_1)\, \bar c^b(x_2)\rangle$

$$= \left(\frac{1}{i}\frac{\delta}{\delta\bar\eta(x_1)}\right)\left(-\frac{1}{i}\frac{\delta}{\delta\eta(x_2)}\right) iW[J,\eta,\bar\eta]\Big|_{J,\eta,\bar\eta=0}$$

$$= i\,\frac{1}{\nabla^2}\cdot\delta^{ab}\,\delta^{(d)}(x_1-x_2)$$

8. Can read off Feynman rules from cubic / quartic terms in the action

$$S = \int d^Dx\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^i A^a_i)^2 - (\partial^i\bar c^a)(D_i c^a)\right]$$

$$= \int d^Dx\left[-\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu\right)^2\right.$$
$$\left. - \frac{1}{2\xi}(\partial^i A^a_i)^2 - (\partial^i\bar c^a)\left(\partial_i c^a - g f^{abc} c^b A^c_i\right)\right]$$

$$\supset \int d^Dx\left[-\frac{1}{2}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)\cdot g f^{abc}A^{b\mu}A^{c\nu}\right.$$
$$\left. - \frac{1}{4}g^2 f^{abc}f^{ade}A^b_\mu A^c_\nu A^{d\mu}A^{e\nu} + g f^{abc}(\partial^i\bar c^a)\,c^b A^c_i\right]$$

$$= \int d^Dx\left[-g f^{abc}\,\partial_\mu A^a_\nu\, A^{b\mu}A^{c\nu}\right.$$
$$\left. - \frac{1}{4}g^2 f^{abc}f^{ade}A^b_\mu A^c_\nu A^{d\mu}A^{e\nu} + g f^{abc}(\partial^i\bar c^a)\,c^b A^c_i\right]$$

The propagators are given by the 2-point functions:


gluon


ghosts

For the 3-point gluon vertex

$$g f^{abc}\left[(p_1-p_3)^\nu g^{\mu\rho} + (p_3-p_2)^\mu g^{\nu\rho} + (p_2-p_1)^\rho g^{\mu\nu}\right]$$

since eg.

$$\mathcal{L} \supset -g f^{abc}(\partial_\alpha A^a_\beta) A^{b\alpha} A^{c\beta} = -g f^{abc}\, i p_1^\alpha A^{a\beta} A^b_\alpha A^c_\beta$$

$$= -g f^{abc} \cdot i p_1^\alpha g^{\beta\rho} A^a_\rho A^b_\alpha A^c_\beta$$

and permutations from the functional derivatives. For the 4-point gluon vertex

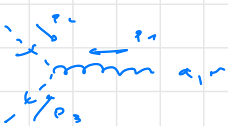
$$-ig^2\left[f^{abe}f^{cde}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) + f^{ace}f^{dbe}(g_{\mu\sigma}g_{\rho\nu} - g_{\mu\nu}g_{\rho\sigma}) + f^{ade}f^{bce}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma})\right]$$

since e.g.

$$\mathcal{L} \supset -g^2 f^{abc} f^{ade} A^b_\alpha A^c_\beta A^{d\alpha} A^{e\beta}$$

$$= -g^2 f^{abc} f^{ade} g^{\alpha\mu} g^{\beta\nu} \cdot A^b_\alpha A^c_\beta A^d_\mu A^e_\nu$$

and permutations from functional derivatives. These have been the same as the Lorentz gauge rules! Now consider the ghost-gluon vertex



$$-g f^{abc} k_3^\mu \delta^{\mu i} \quad \text{(no sum!)}$$

since

$$\mathcal{L} \supset g f^{abc}(\partial^i \bar{c}^a) c^b A^c_i = g f^{abc} \cdot i p_3^i \bar{c}^a A^b_i c^b$$

9. Diagrams that contribute to the gluon 2-point function at 1-loop